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First Semester MCA Degree Examination, June 2012

Discrete Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Define subset, proper subset, power set with an example. **(06 Marks)**

b. Prove the following: i) $A \oplus B = (A \cup B) - (A \cap B)$; ii) $A - (A - B) = A \cap B$ **(06 Marks)**

c. A survey of 500 television watches produced the following information: 285 watch football game, 195 watch hockey game, 115 watch basketball game, 45 watch football and basket ball games, 70 watch football and hockey games, 50 watch hockey and basketball games and 50 do not watch any of the three kinds of games

i) How many people in the survey watch all three kind of games?

ii) How many people watch exactly one of the sports? **(08 Marks)**

2 a. The sample space for an experiment is $S = \{1, 2, 3, 4, 5, 6\}$ where each outcome is equally likely. If event A={1, 2, 3} and event B={2, 4, 6}, determine $P_r(A)$, $P_r(\overline{B})$, $P_r(A \cap B)$, $P_r(A \cup B)$. **(06 Marks)**

b. Prove that : $\sim[\sim[(p \vee q) \wedge r)] \vee \sim q] \Leftrightarrow q \wedge r$ **(06 Marks)**

c. Define tautology and contradiction with an example. Examine whether the following compound proposition is a tautology or a contradiction. $p \wedge [(p \wedge q) \rightarrow r] \rightarrow (q \rightarrow r)$ **(08 Marks)**

3 a. Show that the following argument is valid,

$$\begin{array}{l} p \rightarrow r \\ r \rightarrow s \\ \sim t \vee u \\ t \vee \sim s \\ \sim u \\ \hline \therefore \sim p \end{array}$$

(07 Marks)

b. Define universal and existential quantifier, and the following statement convert to symbolic form and also write its negation. For all x, if x is odd then $x^2 - 1$ is even. **(07 Marks)**

c. Let p(x), q(x) and r(x) be open statements, show that the argument is valid, **(06 Marks)**

$$\begin{array}{c} \forall x[p(x) \rightarrow q(x)] \\ \forall x[q(x) \rightarrow r(x)] \\ \hline \therefore \forall x[p(x) \rightarrow r(x)] \end{array}$$

4 a. Use mathematical induction method, prove that,

$$1.2 + 2.3 + 3.4 + ... + n(n+1) = \frac{n(n+1)(n+2)}{3}$$ **(06 Marks)**

b. Use mathematical induction method to prove that, $\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}$, whenever $A_1, A_2 \ldots A_n$ are subsets of universal set U and $n \geq 2$. **(06 Marks)**

c. Let T(0) = 0, T(1) = 1 and T(n) = $T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + T\left(\left\lfloor \frac{n}{5} \right\rfloor\right)$ for $n \geq 2$. Find T(⌉3). Here $\lfloor a \rfloor$ stands for the greatest integer less than or equal to a real number 'a'. **(05 Marks)**

d. Let $a_0 = 1$, $a_1 = 2$, $a_2 = 3$ and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for all $x \in z^+$, where $n \geq 3$. Find a_6. **(03 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

5 a. Define the Cartesian product of two sets. For any non-empty sets A, B and C. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$ **(05 Marks)**

b. Let A = {1, 2, 3, 4, 5, 6, 7} and R = {<x, y>/x – y is divisible by 3}. Show that R is an equivalence relation. **(05 Marks)**

c. Let A = {1, 2, 3}. Let R and S be the relations on A, whose matrices are

$$M(R) = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \quad M(S) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

then find M(R-1), M(RUS), M(ROS), $M(R^2)$ **(05 Marks)**

d. Define the partition set with an example. Let A={1, 2, 3, 4, 6, 8, 12} and R be the partial ordering on a defined by aRb if a divides b, draw the Hasse diagram of the poset (A, R). **(05 Marks)**

6 a. Define one to one and on to function, also verify the following functions are one-to-one or not, i) f(x) = 3x + 7 ii) $g(y) = y^4 - y,\ \forall x, y \in R$ **(05 Marks)**

b. There are six programmers in the MCA department who can assist ten departments in the college. In how many ways can these departments be assisted by the six programmers so that each is working at least at one department, by using stirling number of second kind. **(06 Marks)**

c. State the Pigeonhole principle. Thirteen persons have first names ‘Darshini, Shree, Ammu” and last names “Raju, Prabhu, Reddy, Murthy”. Show that atleast two persons have the same first and last names. **(05 Marks)**

d. Let f, g, h: R → R, where $f(x) = x^2$, g(x) = x+5 and $h(x) = \sqrt{x^2 + 2}$; then find $f \circ (g \circ h)$ and $(h \circ g) \circ f$. **(04 Marks)**

7 a. Show that any group G is abelian iff $(ab)^2 = a^2 b^2$, for all $a, b \in G$. **(06 Marks)**

b. Define homomorphism and isomorphism. If f: G→H is a homomorphism then prove that $f(a^{-1}) = [f(a)]^{-1}$, for all $a \in G$. **(07 Marks)**

c. Define the encoding function $E : Z_2^3 \to Z_2^6$ by means of the parity-check matrix. **(07 Marks)**

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$. Determine all code words.

8 a. Let $H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$ be the parity-check matrix for a Hamming (7, 4) code.

i) Encode the following messages 1000, 1011, 1110

ii) Decode the following received words 1100001, 0010001 **(10 Marks)**

b. Define Ring and Integral domain. **(05 Marks)**

c. Determine the values of the integer n > 1, for which the given congruence is true

i) $57 \equiv 1 \pmod{n}$ ii) $68 \equiv 37 \pmod{n}$ **(05 Marks)**

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